Rewarding Learning

ADVANCED SUBSIDIARY (AS)

General Certificate of Education

2022

Further Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

SFM11

[SFM11]

MONDAY 16 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 4 & 1 & 0 \\ 3 & -2 & p \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ -2 & 2 \\ 1 & p+1 \end{pmatrix}$$

(i) Calculate the matrix product **AB**, leaving your answer in terms of p. [2]

(ii) Given that **AB** is singular, find the possible values of p. [4]

2 The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 3\mathrm{i} \quad \text{and} \quad z_2 = 1 + q\mathrm{i}$$

where q is a real number.

(i) Find the modulus and argument of z_1 [3]

(ii) Given that $z_1 + 3z_2$ is a real number, find the value of q. [2]

(iii) Hence simplify

$$\frac{z_1}{\overline{z_2}}$$

giving your answer in the form $a + b\mathrm{i}$, where a and b are real numbers. [4]

3 The roots of the quadratic equation

$$2x^2 - 4x + 5 = 0$$

are α and β.

(i) Write down the values of $(\alpha + \beta)$ and $\alpha\beta$. [3]

(ii) By considering $(\alpha + \beta)^3$, find the value of $\alpha^3 + \beta^3$ [4]

(iii) Find a quadratic equation with integer coefficients whose roots are

$$\frac{1}{\alpha + 1} \quad \text{and} \quad \frac{1}{\beta + 1}$$ [6]

4 **(i)** The matrix $\mathbf{S} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

Describe fully the transformation represented by **S** [2]

(ii) The set of points which form the curve

$$x^2 + 10y^2 + 6xy - 25 = 0$$

is transformed by the matrix **S**

Show that the curve formed by the image points is a circle and state its centre and radius. [6]

(iii) The matrix $\mathbf{N} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

Find the single matrix that represents the combined effect of the transformation represented by **S** followed by that represented by **N** [2]

5 Two lines are given by the equations

$$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+4}{3} \quad \text{and} \quad x-3 = \frac{y-2}{-1} = \frac{z+3}{-2}$$

(i) Show that the two lines intersect and find their point of intersection. [9]

(ii) Find the Cartesian equation of the plane which contains both these lines. [6]

(iii) Find the coordinates of the point at which this plane cuts the y-axis. [1]

6 A matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & p \\ 1 & -2 & -2 \\ 0 & p+1 & 2 \end{pmatrix}$$

(i) Show that the determinant of **M** is given by

$$p^2 + 5p - 6$$ [3]

A system of linear equations is given by

$$\begin{aligned} 2x + y + pz &= 3 \\ x - 2y - 2z &= 6 \\ (p+1)y + 2z &= -7 \end{aligned}$$

(ii) Find the possible values of p for which this system has a unique solution. [3]

(iii) If $p = 2$ find, without the use of a calculator, the inverse of **M** [5]

(iv) Hence for $p = 2$, find the unique solution of the system of equations. [4]

7 **(a)** **(i)** Find the real values of a and b such that

$$(a + b\mathrm{i})^2 = 12 - 16\mathrm{i}$$

[6]

(ii) Hence write down the solutions to the equation

$$z^2 = 12 - 16\mathrm{i}$$ [2]

(b) Sketch, on a carefully labelled Argand diagram, the region consisting of points which represent a complex number, z, satisfying both

$$|z - 2\mathrm{i}| \leqslant 4 \quad \text{and} \quad |z + 4 - \mathrm{i}| \leqslant |z + \mathrm{i}|$$ [7]

8 ABCDE is a rectangular-based pyramid with apex E, as shown in **Fig. 1** below.

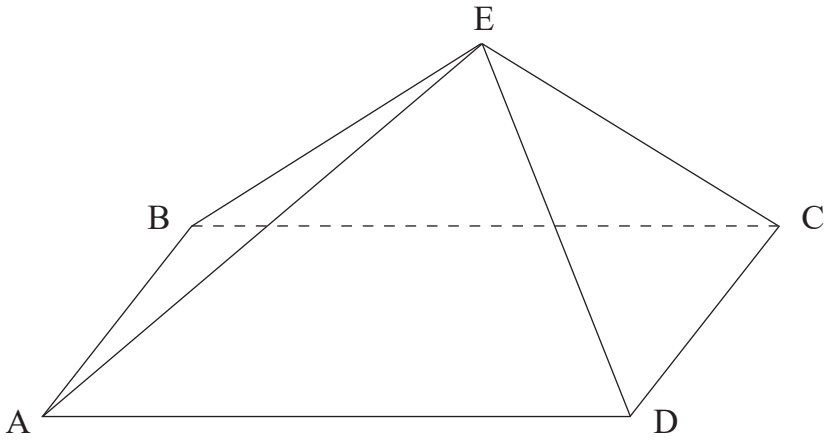

Fig. 1

The vertices A, B, D and E have coordinates (−2, −2, 1), (1, 2, 6), (−3, 0, 0) and (2, −2, 2) respectively.

(i) Find the coordinates of vertex C. [3]

(ii) Find the volume of the pyramid. [7]

The line l has the vector equation

$$\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

(iii) Find the shortest distance between the point E and the line l. [6]

THIS IS THE END OF THE QUESTION PAPER